NOTATION

d = column diameter
D = molecular diffusivity
G = average gas mass flow rate
k_g = gas phase mass transfer coefficient
m = slope of a line in Figure 7, exponent on gas Reynolds number
p = total pressure
p_{BM} = log mean partial pressure of air in the column
N_{Re_g} = gas Reynolds number relative to pipe wall $= \frac{G \cdot d}{\mu_g}$
N_{Re_l} = liquid Reynolds number $= 4\Gamma/\mu_l$
N_{Sc} = average gas phase Schmidt number $= \frac{\mu_g}{\rho_g D_g}$
N_{Sh} = gas Sherwood number $= k_g \frac{RTd}{D_g}$

Greek Letters

μ = viscosity
ρ = density
Γ = mass flow rate of liquid per unit perimeter

Subscripts

g = gas
l = liquid

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Fluid Mechanical Analogies

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The problem of steady, axial, irrotational flow of a Newtonian fluid through a nonporous duct is analogous to several other problems. The knowledge of these analogies was well known thirty to forty years ago; however, apparently, as a result of the separation of the fields of mechanics of fluids and mechanics of elastic solids, these analogies are not as widely known now. Recently, several analytical solutions to the above mentioned fluid mechanics problem have been published and are available from previously published solutions for an analogous problem. These analogies will be briefly reviewed and references to the fundamental work will be given. Examples of recently published solutions to the Newtonian fluid flow problem which are available from previously published solutions for an analogous problem will also be given.

Love (*1*) reviewed and summarized the known analogies. The subject of that discussion is the analogy of the torsion problem, which is the topic of the chapter, to various fluid mechanics problems. The analogy rests on the principle that the twisting force couple causing the torsion can be replaced by a statically equivalent axial compression force. Poisson's equation can be solved by a Green's function, which is specific to the boundary or boundaries of the prism, to yield a plane harmonic function ψ. The function is the conjugate of the torsion function ϕ.

$$\phi + i\psi = f(x + iy)$$

where x and y are position variables.

A solution of Poisson's equation may be obtained if the condition

$$\psi - 1/2\,(x^2 + y^2) = \text{constant}$$

is satisfied at the boundary. The function ψ has been determined for a variety of prism shapes (*1* to *3*).

The function ψ is mathematically identical with the velocity or stream function of three fluid mechanical problems. Two are for frictionless fluids and will not be discussed here. The third analogy is that the function $\psi - 1/2\,(x^2 + y^2)$ is . . . "Mathematically identical with the velocity in a certain laminar motion of a viscous fluid" (*1*) . . . when the motion is caused by a pressure gradient along a duct of the same shape as the twisted prism. The analogy was first observed by Boussinesq (*4*). Application of a known solution of the torsion problem to a fluid mechanics problem is discussed by Bairstow and Berry (*5*) and Dryden et al. (*6*).

The application of this analogy would have obviated the need for recently published solutions for Newtonian fluid flow through an eccentric annular duct (*7* to *9*) and an isosceles triangular duct (*10*). The eccentric annular duct solution previously had been given by Bairstow and Berry (*5*) (see also *6*, p. 198) who utilized the solution to the analogous torsion problem of MacDonald (*11*). The solution for the isosceles triangular duct could be deduced from the analogous torsion solution of Saint-Venant (*3*, pp. 121-122).

In conclusion, it is hoped that this note will lead to economy of effort on the problem of axial, steady, irrotational flow of a Newtonian fluid in ducts. Many solutions to the analogous problem of torsion in a prism of the same shape are available (*1* to *3*). In many cases direct application of these results should be possible.

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